

Quasinormal modes of a Schwarzschild black hole

When a black hole is perturbed, it responds by emitting a characteristic “ringdown” signal: a superposition of damped oscillations known as quasinormal modes (QNMs). These complex frequencies encode the mass and spin of the remnant and play a central role in gravitational-wave astronomy. This project focuses on computing the QNM spectrum of the Schwarzschild black hole using Leaver’s continued fraction method, one of the most accurate techniques available. Specifically, this project has the following sections in mind

1. Derive the master wave equation governing linear perturbations of a Schwarzschild black hole, and carefully formulate the physical boundary conditions: purely ingoing waves at the horizon and purely outgoing waves at infinity.
2. Construct the Frobenius/series solution used in Leaver’s approach, derive the associated recurrence relation, and implement the continued fraction condition whose roots give the complex QNM frequencies.
3. Compute the first few quasinormal mode frequencies for selected multipoles (e.g. $\ell = 2, 3$), benchmark against published values, and explore trends with overtone number. (Optional extension: implement Nollert’s improvement to access high overtones efficiently.)

The student must be comfortable with scientific computing (in any programming language of choice). Some comfort with differential equations, complex arithmetic, and numerical root-finding is helpful. Overall, this project is best suited for students who enjoy eigenvalue problems, careful numerical work, and are curious about the mathematics behind black-hole ringdown.

Some references:

1. E. W. Leaver. An analytic representation for the quasi-normal modes of Kerr black holes. *Proc. R. Soc. Lond. A*, 402:285–298, 1985. doi:10.1098/rspa.1985.0119.
2. H.-P. Nollert. Quasinormal modes of Schwarzschild black holes: The determination of quasinormal frequencies with very large imaginary parts. *Phys. Rev. D*, 47:5253–5258, 1993. doi:10.1103/PhysRevD.47.5253.
3. E. Berti, V. Cardoso, and A. O. Starinets. Quasinormal modes of black holes and black branes. *Class. Quant. Grav.*, 26:163001, 2009. doi:10.1088/0264-9381/26/16/163001, arXiv:0905.2975 [gr-qc].